SOLUTION SET 2

1. First we must calculate the ratio of average O:

\[ \text{Rave} = \frac{0.205 \, \%}{99.757 \, \%} = 0.002055 \]

Then we use Rave as the 'standard' to calculate the 8^\text{th} \text{O} of snow relative to Rave.

\[ 8^\text{th} \text{O snow} = \left( \frac{R_{\text{snow}} - \text{Rave}}{\text{Rave}} \right) \times 1000 = \frac{(0.00205 - 0.002055)}{0.002055} \times 1000 = -24.3 \, \% \]

Now we use snow as the 'standard' and calculate 8^\text{th} \text{O} of PDS:

\[ 8^\text{th} \text{O PDS} = \left( \frac{R_{\text{pds}} - R_{\text{snow}}}{R_{\text{snow}}} \right) \times 1000 = \frac{(0.0020672 - 0.002055)}{0.002055} \times 1000 = +31.022 \, \% \]
\[ \delta^{18}O_{\text{snow}} = \left( \frac{R_{\text{sem}}}{R_{\text{snow}}} - 1 \right) \times 1000 \]

\[ \delta^{18}O_{\text{PDB}} = \left( \frac{R_{\text{sem}}}{R_{\text{PDB}}} - 1 \right) \times 1000 \]

\[ \frac{\delta^{18}O_{\text{snow}}}{1000} = \frac{R_{\text{sem}}}{R_{\text{snow}}} - 1 \]

\[ \frac{\delta^{18}O_{\text{snow}}}{1000} + 1 = \frac{R_{\text{sem}}}{R_{\text{snow}}} \]

\[ \text{snow: } R_{\text{sem}} = \left( \frac{\delta^{18}O_{\text{snow}}}{1000} + 1 \right) \times 0.002005 \]

\[ \text{PDB: } R_{\text{sem}} = \left( \frac{\delta^{18}O_{\text{PDB}}}{1000} + 1 \right) \times 0.0020072 \]

\[ R_{\text{sem}} = R_{\text{sem}} \text{, so: } \]

\[ \left( \frac{\delta^{18}O_{\text{snow}}}{1000} + 1 \right) \times 0.002005 = 0.0020072 \left( \frac{\delta^{18}O_{\text{PDB}}}{1000} + 1 \right) \]

\[ \frac{\delta^{18}O_{\text{snow}}}{1000} + 1 = 1.0310 \left( \frac{\delta^{18}O_{\text{PDB}}}{1000} + 1 \right) \]

\[ \frac{\delta^{18}O_{\text{snow}}}{1000} + 1 = 1.0310 e^{-3} \delta^{18}O_{\text{PDB}} + 1.0310 \]

\[ \frac{\delta^{18}O_{\text{snow}}}{1000} = 1.0310 e^{-3} \delta^{18}O_{\text{PDB}} + 0.0310 \]

\[ \delta^{18}O_{\text{snow}} = 1.0310 \left( \delta^{18}O_{\text{PDB}} \right) + 31.0 \]
Technically, all you need are two points on a $\delta^{18}O_{\text{snow}}$ vs. $\delta^{18}O_{\text{POB}}$ plot. Of course you could do more in Excel. First, calculate $\delta^{18}O_{\text{snow}}$ and $\delta^{18}O_{\text{POB}}$ for two different $R$ values.

1. Terrestrial oxygen $R = 0.002055$

$$R_{\text{snow}} = \left( \frac{0.002055 - 0.0020505}{0.002055} \right) \times 1000 = 24.94\%$$

$$R_{\text{POB}} = \left( \frac{0.002055 - 0.0020492}{0.0020492} \right) \times 1000 = -5.90\%$$

2. $R = 0.002060$ (chosen arbitrarily)

$$R_{\text{snow}} = \left( \frac{0.002060 - 0.002055}{0.002055} \right) \times 1000 = 27.44\%$$

$$R_{\text{POB}} = \left( \frac{0.002060 - 0.0020492}{0.0020492} \right) \times 1000 = -3.48\%$$

If $x = \delta^{18}O_{\text{snow}}$ and $y = \delta^{18}O_{\text{POB}}$, then slope $= \frac{\Delta y}{\Delta x}$

$$\text{Slope} = \frac{-5.90 - (-3.48)}{27.44 - 27.4} = \frac{-2.42}{-2.46} = 0.984$$

And using a point to calculate intercept:

$$\delta^{18}O_{\text{POB}} = 0.984 \left( \delta^{18}O_{\text{snow}} \right) + b$$

$$-3.48\% = 0.984 (27.4\%) + b$$

$$b = 30.44$$

So full equation:

$$\delta^{18}O_{\text{POB}} = 0.984 \left( \delta^{18}O_{\text{snow}} \right) + 30.44$$
If the lake is at steady state, then its $\delta^{18}O$ is NOT changing. Given the inflows, the evaporation must equal the mass and elementary isotopic flux of the inflows. If evaporation did not have this property, then the lake volume and/or isotopic value would change with time.

So $\delta^{18}O_{\text{in}} = \delta^{18}O_{\text{out}}$

$\delta^{18}O_{\text{in}} = 0.9(-16.0) + 0.1(-5.0) = -14.9\%$

$\delta^{18}O_{\text{out}} = -14.9\%$

$R_{\text{out}} = (\frac{\delta^{18}O_{\text{out}}}{1000} + 1) R_{\text{snow}}$

$R_{\text{lake}} = (\frac{\delta^{18}O_{\text{lake}}}{1000} + 1) R_{\text{snow}}$

$\alpha = \frac{R_{\text{out}}}{R_{\text{lake}}} = \frac{\delta^{18}O_{\text{out}}}{1000} + 1 = 0.9851$

$\frac{\delta^{18}O_{\text{lake}}}{1000} + 1 = 0.994$

$0.9940 = \alpha$

NOTE: You could also use the relationship that:

$\delta_e - \delta_v = \Delta_e-v = 1000 \ln \alpha - v$

The key is still to realize that $\delta^{13}O$ vapor = -14.9\%.
a) \( S^{18}O_{qtz} - S^{18}O_{w} = 3.38 \ \frac{10^6}{T^2} - 2.90 \)

200°C = 473 K

\( S^{18}O_{qtz} = 12.21 \% \text{ SMOW} \)

\( S^{18}O_{cal} - S^{18}O_{w} = 2.98 \ \frac{10^6}{T^2} - 2.89 \)

300°C = 573 K

\( S^{18}O_{cal} = 5.58 \% \text{ SMOW} \)

b) The key here is to realize that for quartz and calcite to have the same \( S^{18}O \), the quartz must have precipitated at a higher temp and for a slightly longer.

Let's first calculate a reasonable \( S^{18}O_{cal} \) for a hydrothermal calcite:

\[ T = 300°C, \quad S^{18}O_{w} = 0 \% \]

\( S^{18}O_{cal} = +5.58 \% \text{ SMOW (see above)} \)

For this same \( S^{18}O_{w} \) and \( T \),

\( S^{18}O_{qtz} = +7.39 \% \text{ SMOW} \)

Let's increase \( T \) by 100°C, and calculate the \( S^{18}O_w \) necessary to obtain a \( S^{18}O_{qtz} \) of +5.58 % SMOW

\[ T = 400°C = 673 K \]

\[ \Delta_{qtz-w} = 3.38 \left( \frac{10^6}{(673)^2} \right) - 2.90 \]

\[ \Delta_{qtz-w} = +4.56 \]

\( S^{18}O_{qtz} - S^{18}O_{w} = +4.56 \)

\[ 5.58 - S^{18}O_{w} = +4.56 \]

\[ S^{18}O_{w} = 1.02 \% \text{ SMOW, } T = 673 K \]


<table>
<thead>
<tr>
<th>Meteorite</th>
<th>Feldspar $\delta^18$O</th>
<th>Pyroxene $\delta^18$O</th>
<th>Olivine $\delta^18$O</th>
<th>Anorthite %</th>
<th>$\beta$</th>
<th>$\Delta$ pla-pyr</th>
<th>$\Delta$ pyr-oli</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allegan</td>
<td>5.12</td>
<td>4.34</td>
<td>3.4%</td>
<td>12</td>
<td>0.12</td>
<td>0.78</td>
<td>0.65</td>
</tr>
<tr>
<td>Estacado</td>
<td>5.75</td>
<td>4.36</td>
<td>3.8%</td>
<td>12</td>
<td>0.12</td>
<td>1.41</td>
<td>0.54</td>
</tr>
<tr>
<td>Bjurbol</td>
<td>7.70</td>
<td>5.16</td>
<td>4.3%</td>
<td>11</td>
<td>0.11</td>
<td>1.83</td>
<td>1.43</td>
</tr>
<tr>
<td>Badger</td>
<td>6.20</td>
<td>5.18</td>
<td>4.2%</td>
<td>11</td>
<td>0.11</td>
<td>1.11</td>
<td>0.85</td>
</tr>
<tr>
<td>Moos</td>
<td>6.23</td>
<td>4.95</td>
<td>4.1%</td>
<td>10</td>
<td>0.1</td>
<td>1.28</td>
<td>0.85</td>
</tr>
<tr>
<td>Modoc</td>
<td>6.17</td>
<td>5.12</td>
<td>4.0%</td>
<td>10</td>
<td>0.1</td>
<td>1.07</td>
<td>1.02</td>
</tr>
<tr>
<td>Shaw</td>
<td>5.75</td>
<td>5.03</td>
<td>4.2%</td>
<td>10</td>
<td>0.1</td>
<td>0.7</td>
<td>0.33</td>
</tr>
<tr>
<td>Olivenza</td>
<td>6.27</td>
<td>5.16</td>
<td>4.3%</td>
<td>10</td>
<td>0.1</td>
<td>1.11</td>
<td>0.83</td>
</tr>
<tr>
<td>St. Severn</td>
<td>6.58</td>
<td>5.31</td>
<td>4.2%</td>
<td>10</td>
<td>0.1</td>
<td>1.27</td>
<td>1.04</td>
</tr>
</tbody>
</table>

**Concordia plot for mineral pairs**

Calculate concordia line for mineral pairs, calculated from equations given in problem set:

$$\Delta \text{ pyr-oli} = \beta \Delta \text{ pla-pyr}$$

$\Delta$ pyr-oli = $0.6122x + 0.1435$

$R^2 = 0.4429$

Colored lines = concordia

<table>
<thead>
<tr>
<th>Temp (°C)</th>
<th>$\beta$=0.10</th>
<th>$\beta$=0.11</th>
<th>$\beta$=0.12</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>11.47137</td>
<td>11.39662</td>
<td>11.32187</td>
</tr>
<tr>
<td>200</td>
<td>7.13363</td>
<td>7.087146</td>
<td>7.040651</td>
</tr>
<tr>
<td>300</td>
<td>4.360978</td>
<td>4.293962</td>
<td>4.244527</td>
</tr>
<tr>
<td>400</td>
<td>3.523731</td>
<td>3.450769</td>
<td>3.447808</td>
</tr>
<tr>
<td>500</td>
<td>2.671</td>
<td>2.653595</td>
<td>2.63919</td>
</tr>
<tr>
<td>600</td>
<td>2.59413</td>
<td>2.580438</td>
<td>2.56642</td>
</tr>
<tr>
<td>700</td>
<td>1.85804</td>
<td>1.74819</td>
<td>1.663834</td>
</tr>
<tr>
<td>800</td>
<td>1.486224</td>
<td>1.377191</td>
<td>1.368158</td>
</tr>
<tr>
<td>900</td>
<td>1.018944</td>
<td>1.152385</td>
<td>1.144827</td>
</tr>
<tr>
<td>1000</td>
<td>0.594864</td>
<td>0.978445</td>
<td>0.972028</td>
</tr>
<tr>
<td>1100</td>
<td>0.846626</td>
<td>0.84111</td>
<td>0.835593</td>
</tr>
<tr>
<td>1200</td>
<td>0.735576</td>
<td>0.730783</td>
<td>0.725969</td>
</tr>
<tr>
<td>1300</td>
<td>0.645023</td>
<td>0.64082</td>
<td>0.636817</td>
</tr>
<tr>
<td>1400</td>
<td>0.570218</td>
<td>0.566502</td>
<td>0.562767</td>
</tr>
<tr>
<td>1500</td>
<td>0.50771</td>
<td>0.504401</td>
<td>0.501093</td>
</tr>
</tbody>
</table>

The scatter in the datapoints indicate that these minerals did not precipitate in isotopic equilibrium. Pure isotopic equilibrium would result in the mineral pair fractionations falling on a single straight line.

The datapoints indicate a range of temps from 600-1400, an estimate obtained by looking at range of $D$ values (~0.6 to 2) for the pla-pyr pair, and comparing them to the calculated values at various temperatures (table above).
First we need to calculate $\Delta^{18}O$ from the data:

$\delta^{18}O = -0.91 \%$  $\delta^{18}O = -2.01 \%$

$\Delta^{18}O = \delta^{18}O - 0.52 (\delta^{18}O)$

$\Delta^{18}O = 0.1352$

$= 135.2 \text{ per mwg}$

We have loss of O$_2$ to the atmosphere, and gain of O$_2$
by biological production. These must balance in steady state.
We can use isotopic mass balance to determine production,
or O$_2$ (in):

$O_2$ (out): 6.25 mol/m$^2$/day  $\times$ 135.2 per mwg

$O_2$ (out) = 33.75 mg per mwg  mol/m$^2$/day

$O_2$ (in): 33.75 mg per mwg/m$^2$/day mol

$\rightarrow \frac{155 \text{ per mwg}}{155 \text{ per mwg}} = 0.21 \text{ mol/m}^2/\text{day}$

$= \text{Production}$

b) We used $P_{\text{max}}$ and $P_{\text{min}}$, which means we used $\Delta^{18}O_{\text{max}}$ and $\Delta^{18}O_{\text{min}}$.

$\Delta^{18}O_{\text{max}} = \delta^{18}O_{\text{max}} - 0.52 \delta^{18}O_{\text{min}}$

$= -0.82 - 0.52(-2.04)$

$= 240.8 \text{ per mwg}$

$\Delta^{18}O_{\text{min}} = -1.0 - 0.52(-2.04) = 30 \text{ per mwg}$

$P_{\text{max}} = 0.39 \text{ mol/m}^2/\text{day}$

$P_{\text{min}} = 0.048 \text{ mol/m}^2/\text{day}$

c) You might expect to see a diurnal cycle in $\delta^{18}O$, perhaps driven by evaporation and/or rainfall. You would also expect to see a diurnal cycle in $\delta^{18}O$ (whatever affects $\delta^{18}O$ affects $\delta^{18}O$). Also, daytime production should generate a diurnal $\Delta^{18}O$ signal.
What: $\Delta$ at $C_{in} = C_{atm} = 150$ ppm?

There is no diffusion in this case, so $\Delta = -29.4\%$.

Furthermore, we know that there is another endmember, at very high $C_{out}$, that diffusion will dominate, and $\Delta$ will be $-4.4\%$.

So your $\Delta$ values must lie between these two extremes.

**Given:** diffusion goes as the function of $\frac{C_{in}}{C_{out}}$.

**Task:** write an equation that satisfies the no-diffusion case, given that $\Delta = -4.4\%$ when $\frac{C_{in}}{C_{out}}$ goes to zero, and $\Delta = -29.4\%$ when $\frac{C_{in}}{C_{out}} = 1$.

We know that diffusion will act on the difference between the two enrichment factors ($-29 - (-4.4)$), so we have:

$$-29.4 = \left(\frac{150\text{ ppm}}{150\text{ ppm}}\right)(-29 + 4.4) + x$$

$x = -4.4$

So the proposed equation is:

$$\Delta = \left(\frac{C_{in}}{C_{out}}\right)(-29.4 + 4.4) - 4.4$$

This also satisfies the diffusion-limited case!

$C_{out} = 360 \text{ ppm}$, $\Delta = \frac{150}{360}(-29.4 + 4.4) - 4.4 = -14.82\%$. 