FEEDBACKS BETWEEN WEATHERING AND ATMOSPHERIC CO₂ OVER THE LAST 100 MILLION YEARS

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ABSTRACT. This paper examines several critical uncertainties in long-term carbon cycle modeling: specifically, the dependence of weathering rate on temperature and pCO₂, the connection between pCO₂ in the atmosphere and in soils, and the effects of a variable terrestrial biosphere on weathering rates. A balance between weathering and metamorphism is used to compare the paleo-CO₂ levels and paleotemperatures derived from the weathering systems developed by Berner, Lasaga, and Garrels (1983) and Walker, Hays, and Kasting (1981). The results differ primarily because the latter system explicitly separates the dependence of bicarbonate-ion concentration in river waters on temperature and pCO₂. This weathering system has been extended by expressing terrestrial productivity as a function of atmospheric pCO₂ in a Michaelis-Menton equation. When the asymptote of maximum productivity is set at 2, 4, and 8 times the present productivity, the global temperature rise from geophysical forcing on the CO₂-greenhouse is lower by about 1°C, 2°C, and 3°C, respectively, than the temperature rise when productivity is held constant. These calculations delineate the possible scope of the terrestrial biosphere’s capacity to moderate, but not perfectly regulate, climatic changes through its direct effect on soil pCO₂.

INTRODUCTION

Walker, Hays, and Kasting (1981) introduced a system of relatively simple algebraic expressions to balance the removal of atmospheric pCO₂ by weathering with its supply by volcanic outgassing. They showed that the response of the CO₂-greenhouse to forcing from an increasing solar flux can be reduced by increased weathering rates associated with the global surface temperature rise. Berner, Lasaga, and Garrels (1983, hereafter known as BLAG) created an important, time-dependent, and substantially more complex model for the carbonate-silicate geochemical cycle by linking a system of reservoirs for lithospheric dolomite, calcite, and silicates; oceanic magnesium, calcium, and bicarbonate ions; and atmospheric pCO₂ and globally-averaged surface temperature. They found that the most significant influences on climate were seafloor spreading rate and continental area.

How suitable are simple steady-state formulations compared to the complete BLAG model for calculating paleo-CO₂ and paleoclimate? If adequate, the simple systems have the advantage of facilitating examinations of the sensitivity to alternative formulations for the key processes, such as the CO₂-greenhouse effect and the bicarbonate ion concentration in continental river runoff. The effect of unexamined, new factors, such as vegetation, which Lovelock and Whitfield (1982) suggested
could exert a negative feedback on changes in CO$_2$ through the weathering rate, might be best initially formulated and explored through use of the simple systems. This paper seeks to quantify these issues.

THE STEADY-STATE APPROXIMATION OF THE BLAG RESULTS

By employing a steady-state assumption for the atmosphere and ocean to set initial reservoir sizes for the BLAG model at 100 my ago, Kasting (1984, eq A7) derived an important equation, the terms of which have been modified slightly to fit the specific present purposes:

$$f_{WR} f_A = f_{SR} f_{DC}$$

(1)

The terms $f_{WR}$, $f_A$, and $f_{SR}$ are non-dimensional ratios of the weathering rate per unit area, total continental area, and global seafloor spreading (generation) rate to their present values, respectively. The fourth factor, $f_{DC}$, derives from possible differences in the susceptibility of the dolomite and calcite reservoirs to metamorphic decarbonation. Essentially, eq (1) expresses a balance in the global ocean-atmosphere CO$_2$ system between the weathering sink, which is a product of rate and area, and the metamorphic source, which is a product of plate-tectonic energetics and susceptibility to decarbonation. This equation must hold over scales somewhat longer than the time constant of approx $10^5$ yr for CO$_2$ in the combined ocean-atmosphere system (Kasting, 1984).

Kasting and Richardson (1985) and Kasting and others (1986) found this equation valuable as an analytical tool to provide insight into specific questions raised with BLAG-type models. BLAG (see their eq 62) was also aware of the importance of a steady-state approximation, but it was after considering the ocean in a quasi-steady state that Kasting (1984) deduced the particularly simple form exhibited by eq (1). This simple balance, used by others as an aid in analysis, was never itself the focus. As will be shown, eq (1) deserves to be forwarded as the fundamental explanation of the general BLAG results.

From geophysical data BLAG specified the factors $f_{SR}$ and $f_A$; this study will take the same values for these factors. Furthermore, BLAG showed that the effect of the dolomite and calcite reservoir changes was minor; therefore this study will hold $f_{DC}$ constant and equal to 1. BLAG calculated the remaining term, $f_{WR}$, from a system of equations that link atmospheric pCO$_2$ ($P_{atm}$) to global average values for river runoff per unit area ($R$), temperature ($T$), and river bicarbonate ion concentration ($C$). For later comparison to the analogous formulations of Walker, Hays, and Kasting (1981), the BLAG system is repeated here:

$$\frac{R}{R_o} = 1 + 0.038(T - T_o)$$

(2A)

$$\frac{C}{C_o} = 1 + 0.049(T - T_o)$$

(2B)
$T - T_o = 2.88 \ln \left( \frac{P_{atm}}{P_{atm,o}} \right)$ (2c)

The subscript-o signifies present-day values. Since $f_{WR}$ is defined by

$$f_{WR} = \frac{R}{R_o} \frac{C}{C_o}$$ (3)

it is possible to compute $P_{atm}$ as a function of the other factors in eq (1). Therefore, setting $f_{DC} = 1$, and combining eqs (1), (2A–C), and (3):

$$\frac{P_{atm}}{P_{atm,o}} = \exp \left[ -8.077 + \left( 1.1334 + 64.1 \frac{f_{SR}}{f_A} \right)^{0.5} \right]$$ (4)

With $P_{atm}$ known, $T$ is given by eq (2c). Or, combining eqs (2c) and (4) and approximating the square-root term:

$$T \approx T_o + 23.3 \left[ \left( \frac{f_{SR}}{f_A} \right)^{0.5} - 1 \right]$$ (5)

The temperature deviations, $T - T_o$, using eq (5) differ from those using eq (2c) instead by only several percent over ranges of $f_{SR}/f_A$ typical of BLAG. Note the importance of the ratio $f_{SR}/f_A$ in both eqs (4) and (5). The four different spreading rate scenarios and the continental area-change scenario from BLAG produce four scenarios of this geophysical forcing ratio. The four scenarios are shown in figure 1 for the last 100 my.

With BLAG’s values for $f_{SR}/f_A$, $P_{atm,o}$ and $T_o$, the calculation for $P_{atm}$ and $T$ proceeds from eqs (4) and (5); the results are shown in figures 2 and 3 for the four scenarios.

As figure 2 shows, eq (4) indeed captures the essence of the BLAG results for atmospheric pCO$_2$. Furthermore, if one’s primary interest is in the climatic implications of global seafloor spreading rate and/or continental area, then figure 3 demonstrates that eq (5) is a suitable simplified alternative to running the full BLAG model. Since this steady-state approximation elucidates the specific quantitative control exerted by $f_{SR}$ and $f_A$, it is important to those interested in the BLAG model but who do not have the time or the necessity to run the full model themselves, and it could help examine the consequences of alternatives, such as using volcanic rock abundances as an indicator of metamorphism instead of seafloor spreading (Berner and Barron, 1984).

The “improved” BLAG model of Lasaga, Berner, and Garrels (1985) added an organic carbon reservoir to the geochemical system. It would be possible in principle to “improve” eq (1) by adding terms for the burial and oxidation of organic carbon. But the calculation of these additional CO$_2$ source and sink would require running a time-dependent isotope model as in the work of Lasaga and others. The quantitative
Fig. 1. Four plots of the long term carbon cycle's major geophysical forcing: the ratio of seafloor generation rate to continental area, $f_{SR}/f_A$. The four scenarios for $f_{SR}$ are from BLAG (fig. 4) and are described in BLAG as: A, Pitman; B, corrected Southam and Hay; C, linear; and D, constant. The scenario for $f_A$ is also from BLAG (fig. 2). Since $f_{SR}$ in curve D is constant, this curve isolates the term $f_A^{-1}$.

Fig. 2. A comparison of the evolution of atmospheric CO$_2$ from (a) BLAG's full geochemical cycles model and (b) the steady-state approximation, specifically, eq (4). The four cases in each panel correspond to the four scenarios of the $f_{SR}/f_A$ forcing shown in figure 1.
Fig. 3. A comparison of the evolution of global temperature from (A) BLAG's full geochemical cycles model and (B) the steady-state approximation, specifically, eq (5). The four cases in each panel correspond to the four scenarios of the $f_{SR}/f_A$ forcing shown in figure 1.

differences between the results of the original BLAG and their "improved" model are about the same as those calculated later in this paper with a variable global productivity.

Finally, what about the factor $f_{DC}$? The curious fact that eqs (4) and (5) calculate a $P_{atm}$ and T that are everywhere less than the BLAG values in all cases of figures 2 and 3 has to do with $f_{DC}$. From Kasting's (1984) derivation the value of $f_{DC}$ is

$$f_{DC} = \frac{2 k_{M_{D,\alpha}} M_D + k_{M_{C,\alpha}} M_C}{2 k_{M_{D,\alpha}} M_{D,\alpha} + k_{M_{C,\alpha}} M_{C,\alpha}}$$

(6)

The variation of $f_{DC}$ can be computed from BLAG's values for the metamorphic rate coefficients ($k_{M_{D,\alpha}}$ and $k_{M_{C,\alpha}}$) and the time-dependent masses of dolomite ($M_D$) and calcite ($M_C$). At 100 my ago $f_{DC}$ was approximately equal to 1.07 and then decreased nearly linearly in forward time to 1.00. Eq (1) shows that a higher $f_{WR}$, and therefore $P_{atm}$ and $T_c$ are required if $f_{DC}$, rather than equal to, is greater than 1. But more importantly the variation in $f_{DC}$ between 1.07 and 1.00 is much smaller than the variation in $f_{SR}/f_A$ that occurs for all four spreading rate scenarios (see fig. 1). The analytic, steady-state approximation quantifies the relative importance of the forcing factors that BLAG identified and facilitates focus upon alternative formulations of the crucial processes.

**Alternate Weathering Formulations**

Walker, Hays, and Kasting (1981, hereafter known as WHAK) developed expressions relating global runoff, average river bicarbonate concentration, global temperature, and atmospheric CO$_2$. These
expressions constitute an alternate system to that of BLAG for calculating weathering. Following the same format used for the BLAG system in eqs (2A-C), the three analogous equations from WHAK are:

\[
\frac{R}{R_o} = \exp \left( \frac{T - T_o}{60} \right) \quad (7A)
\]

\[
\frac{C}{C_o} = \left( \frac{P^*}{P_o^*} \right)^{0.3} \exp \left( \frac{T - T_o}{17.7} \right) \quad (7B)
\]

\[
T - T_o = 4.6 \left( \frac{P_{atm}}{P_{atm,0}} \right)^{0.564} - 4.6 \quad (7C)
\]

The interested reader should refer to the original papers of WHAK and BLAG for the detailed arguments behind these various parameterizations. The runoff and CO₂-greenhouse equations of the two systems have the identical variable dependence and can be directly compared (eqs 2A and 7A; eqs 2C and 7C). The first term of the Taylor series for the exponential term of eq (7A) shows that WHAK’s runoff \((R/R_o \approx 1 + 0.0167(T - T_o))\) is less than half as sensitive to \(T\) as BLAG’s. Figure 4 compares eqs (2C) and (7C) for the CO₂-greenhouse, respectively, from BLAG and WHAK for \(P_{atm} = P_{atm,0}\) to \(30 P_{atm,0}\). Within this range the two are comparable; the two weathering systems will be used only with the Southam and Hay spreading curve, which yields \(P_{atm}\) values within this range.

![Graph](image-url)

**Fig. 4.** A comparison of the CO₂-greenhouse developed by BLAG and WHAK, eqs (2C) and (7C), respectively. At higher atmospheric CO₂ levels than shown the two diverge significantly.
The river bicarbonate ion equations for BLAG and WHAK, on the other hand, contain a major conceptual difference. In eq (2B), C is an explicit function of T only; in addition to T, eq (7B) includes an explicit term for CO₂ partial pressure, which I have marked with an asterisk (P*). For although WHAK took the pressure term in eq (7B) to represent P_{atm}, other interpretations are possible. (Note that P_{atm} in eq (7C), the CO₂-greenhouse equation, is most definitely atmospheric pCO₂.)

The P* term in eq (7B) has three alternative interpretations. The first is that P* does indeed mean P_{atm}. Then, by combining eqs (3), (7A), and (7B), I form the first of three WHAK weathering formulations:

$$f_{WR} = \left( \frac{P_{atm}}{P_{atm,0}} \right)^{0.3} \exp \left( \frac{T - T_o}{13.7} \right)$$  \hspace{1cm} (8A)

Interestingly, BLAG specifically stated that the T-dependence of C in eq (2B), taken from North American data, is in fact due to soil microbial activity being positively correlated with regional temperature. This activity produces CO₂, and thus T could be a proxy for soil pCO₂. In contrast, the separate T- and P-dependencies of eq (7B) came from laboratory experiments (in the absence of biota) that evaluated both temperature and CO₂ pressure effects separately at high CO₂ pressures and temperatures, 2 to 20 bars and 100° to 200°C, respectively. The assumed extrapolation to lower pressures was not verifiable, but river data (different than the BLAG river data) did validate the T-dependence. An expansion of the exponential in eq (7B) yields C/C₀ ≈ 1 + 0.056(T - T₀), which, for sufficiently small values of (T - T₀), is very close to its BLAG counterpart in eq (2B). Berner and Barron (1984) were, in fact, able to verify the BLAG T-dependence of C, which is proportional to silicate concentration, for silicate weathering by up-to-date river water data. WHAK suggested that the river data's T-dependence may be "contaminated" by CO₂ from biological activity: BLAG definitely had this viewpoint.

Since the T-dependence of bicarbonate in WHAK may already subsume a microbial-activity-driven P-dependence, the T-term of eq (7B) could arguably stand alone. Then eq (7B) becomes C/C₀ = \exp[(T - T₀)/17.7], which in combination with eqs (3) and (7A) yields a second WHAK weathering factor formulation:

$$f_{WR} = \exp \left( \frac{T - T_o}{13.7} \right)$$  \hspace{1cm} (8B)

The third possibility comes from considering the P* term explicitly as soil pCO₂ (P_{soil}) generated primarily by the respiration of microbes and plant roots: P_{soil} is typically 10 to 100 times P_{atm} (Holland, Lazar, and McCaffrey, 1986). If P* in eq (7B) is P_{soil}, the final case for f_{WR} can be written as
\[ f_{WR} = \left( \frac{P_{soil}}{P_{soil,o}} \right)^{0.3} \exp \left( \frac{T - T_o}{13.7} \right) \]  

(8c)

To use eq (8c) an expression is needed for \( P_{soil} \). In a steady-state, the belowground respiration is at least equal to the belowground net primary production, which is a significant fraction of the total net primary production. For example, this fraction can be as high as 80 percent in grasslands (Milchunas and others, 1985). The global silicate weathering rate in BLAG removes \( 11.48 \times 10^{18} \) moles of CO\(_2\) from the atmosphere per my, or 0.14 GtC/yr, but this removal in actuality takes place from the soil's air. Green plants annually fix 100 to 120 GtC (Olson, 1982), and therefore on the order of tens of GtC as CO\(_2\) is released into the soil's air by the respiration of roots and the respiration of microbes living on roots, animal products, and aboveground production that enter the soil as detritus. Since this belowground CO\(_2\) release is so much greater than the weathering sink, the upward diffusion of CO\(_2\) from the soil into the atmosphere must be the major sink for soil CO\(_2\) to balance the belowground production of CO\(_2\) by the biota.

In the same spirit as the global averages for \( T, R, \) and \( C \), we can define a total (aboveground and belowground) productivity (\( \Pi \)), a diffusive exchange coefficient (\( k_{soil} \)) between the “boxes” of atmosphere and soil, and a fraction of the productivity (\( f_{root} \)) released by the belowground ecology as soil CO\(_2\). In an annually-averaged, approximate steady-state, the flux of CO\(_2\) from the soil to the air balances its production in the soil:

\[ k_{soil} (P_{soil} - P_{atm}) = f_{root} \Pi \]  

(9)

The terms \( k_{soil} \) and \( f_{root} \) are assumed constant, and therefore their ratio can be calculated from present conditions:

\[ \frac{f_{root}}{k_{soil}} = \frac{P_{soil,o} - P_{atm,o}}{\Pi_o} \]  

(10)

By combining eqs (9) and (10) and solving for \( P_{soil} \), the value for \( P*/P* = P_{soil}/P_{soil,o} \) for use in eq (8c) to close the third WHAK weathering formulation is:

\[ \frac{P_{soil}}{P_{soil,o}} = \frac{\Pi}{\Pi_o} \left( 1 - \frac{P_{atm,o}}{P_{soil,o}} \right) + \frac{P_{atm}}{P_{soil,o}} \]  

(11)

Eq (8A-c) constitutes 3 cases for the WHAK weathering rate, and each in combination with eqs (1) and (7c) can calculate \( P_{atm} \) and \( T \) for 100 my. As before, \( f_{PC} \) will be equal to 1, and the ratio \( f_{SR}/f_A \) is shown in figure 1. The case of eq (8c) additionally requires eq (11), and for now productivity is held constant (\( \Pi = \Pi_o \)). Also, (\( P_{soil,o}/P_{atm,o} \)) is taken equal to 10. Figure 5, A and B, shows \( P_{atm} \) and \( T \) for these three WHAK cases in comparison to the results from the steady-state BLAG approximation of
eqs (4) and (5) for the corrected Southam and Hay spreading rate scenario. Several major points stand out.

The first WHAK case of eq (8A), as shown by curves W1 in figure 5, produces significantly different results for $P_{\text{atm}}$ and $T$ than the BLAG system. Taking $P^* = P_{\text{atm}}$, which means $C/C_0 \propto (P_{\text{atm}}/P_{\text{atm},0})$, gives a strong negative-feedback via weathering to the $f_{\text{SR}}/f_A$ forcing, resulting in a much cooler Eocene and Cretaceous than given by the steady-state BLAG approximation. For example, while eq (5) calculates a temperature of $24^\circ \text{C}$ at 90 my ago, the system of eqs (1), (7C), and (8A) calculates $18^\circ \text{C}$.

In contrast, as shown by curve W2 in figure 5b, the second WHAK case of eq (8B) produces paleotemperatures remarkably close to the steady-state BLAG approximation, labeled “eq (5)” in the figure. This similarity demonstrates that the weathering and $\text{CO}_2$-greenhouse systems of BLAG and WHAK give virtually identical results for paleotemperatures if the $P^*$-term in eq (7B) is eliminated.
The third case — eqs (8c) and (11) — as shown by curves W3 of figure 5, A, B, results in $P_{atm}$ and $T$ values in between those of the other two cases. The results must be between as determined by the variations of $P*/P_0^*$ as a function of $P_{atm}$. For case 1, $P*/P_0^* = P_{atm}/P_{atm,0}$ and $d(P*/P_0^*)/dP_{atm} = P_{atm,0}^{-1}$. In case 2 we can consider $P*/P_0^* = 1$, so $d(P*/P_0^*)/dP_{atm} = 0$. In the third case, $P*/P_0^* = P_{soil}/P_{soil,0}$, and by eq (11), $d(P*/P_0^*)/dP_{atm} = P_{soil,0}^{-1}$ if $\Pi = \Pi_0$. Because $P_{soil,0}^{-1}$ is generally much less than $P_{atm,0}^{-1}$, the buffering explicitly due to pressure in case 3 will be less than case 1 but more than case 2. Compared to curve W2 in figure 5, curve W3 has a 3°C lower temperature about 90 my ago, a significant difference from BLAG's Cretaceous, but even lower temperatures are produced by W1. The W3 curves are dependent upon the value for $P_{soil,0}$; larger values for $P_{soil,0}$ would move the results closer to the W2 curves. But because the seasonal behavior of $P_{soil}$ and its depth-dependence is complex (Buyanovsky and Wagner, 1983), an average $P_{soil}$ is not well-determined. The major caveat is the assumption of constant productivity, $\Pi$.

**CALCULATION OF A FEEDBACK BETWEEN PRODUCTIVITY AND CO$_2$**

Terrestrial productivity would probably not be constant with differing $P_{atm}$. As yet there is no consensus what the specific global impact of our ever-increasing atmospheric pCO$_2$ levels will be on the ecosystems of the future, but all indications from the existing data on the response of leaves and whole plants point to a world with a higher $\Pi$ (DOE, 1985). Presumably one result of a more productive biosphere would be increased belowground respiration that would pump soil pCO$_2$ to a higher level (Berner and Barron, 1984).

Streams in Iceland's vegetated regions have bicarbonate values 2 to 3 times higher than streams running through the barren areas (Cawley, Burruss, and Holland, 1969). According to eq (7b), if vegetation causes $P_{soil}$ to increase by 10 to 100 times, C will increase by $(10)^{0.5}$ to $(100)^{0.5}$, or 2.0 to 4.0 times. Since these estimated values for the increase in C are consistent with the Iceland data, the WHAK system will be adopted to examine the effect of a variable $\Pi$ in the $P_{atm}$ and $T$ calculation.

A number of biological and environmental factors could operate simultaneously to increase $\Pi$ with increasing $P_{atm}$. Raising atmospheric pCO$_2$ above current levels, for example, increases growth in all the ubiquitous C$_3$ species tested so far by increasing the photosynthetic rate, the carbon fixed per area per time, which is related to the quantum efficiency, the carbon fixed per photosynthetically active photon (DOE, 1985). This “fertilization” primarily works by lowering the loss rate of the initial photosynthetic products during photorespiration (Pearce and Bjorkman, 1983; Tenhunen, Hesketh, and Harley, 1980).

Also, at higher $P_{atm}$'s the plant's leaf pores, the stomates, contract, which reduces the transpiration rate. If it takes less water to grow the same amount of plant biomass — typically 100's of kg H$_2$O per kg dry matter — more biomass could grow from a given amount of water; this
higher "water utilization efficiency" could have global consequences (Kimball, 1985).

In addition to this "metabolic" water-effect, both eqs (2A) and (7A) show an "environmental" water-effect, namely, the increased global supply of water from the oceans to the continents in a world with higher $P_{\text{atm}}$. Although this effect would not exist in regions such as rainforests, over much of the world an increased water supply would raise productivity because the globally-averaged rain falls in the regime where productivity is still strongly affected by rain in Leith's (1975) empirical productivity/rain relation.

The lengthening of the growing season from the temperature increases in eqs (2c) and (7c) could enhance productivity in the higher latitudes. A reduction in the continental ice sheets ultimately would add productive land the size of Greenland and Antarctica.

A final possible effect comes from the feedback between plants and rainfall. The continents in a GCM study (Shukla and Mintz, 1982) had about 3 times as much rain in the wet-soil case compared to the dry-soil case, these cases being idealizations of respective worlds with and without a terrestrial biosphere. An increased flux of water between the soil and atmosphere might increase the weathering reactions even at a given runoff.

These effects for higher $CO_2$ — increases in the quantum yield, the water utilization efficiency, the continental water supply, the growing season and arable land, and, in addition, a possible transpiration-rainfall feedback — have large individual uncertainties on both regional and global scales. For example, the tropical rainforests, among the most productive regions, are probably already at a maximum productivity limited by nutrient cycles. The intention here is to quantify various possibilities of a $CO_2$-productivity feedback upon weathering, and therefore this work assumes only that these effects will act as a system to increase $\Pi$ as a function of $P_{\text{atm}}$. The Michaelis-Menton equation, which is generally useful in describing the effects of $CO_2$ on plant growth (Allen and others, 1987), is the specific functional form assumed here. This function can be defined uniquely when the dependent variable has an asymptotic maximum and when the half-saturation constant (the independent variable's value when the dependent variable is half its maximum) can be specified. Plant growth as a function of many different limiting factors, such as water, light, $pCO_2$, and nutrients takes this shape (Lemon, 1983). Therefore, writing for $\Pi$:

$$\Pi = \Pi_{\text{max}} \frac{P_{\text{atm}} - P_{\text{min}}}{P_{1/2} + (P_{\text{atm}} - P_{\text{min}})}$$

(12)

In eq (12), $\Pi_{\text{max}}$ is the maximum productivity and $P_{1/2}$ is the value at which $\Pi = 0.5 \Pi_{\text{max}}$. The term $P_{\text{min}}$ is the value of $P_{\text{atm}}$ at which the rate of carbon fixation just balances the photorespiration; if $P_{\text{atm}}$ is less than $P_{\text{min}}$, then $\Pi = 0$. By requiring that eq (12) returns $\Pi_o$ when $P_{\text{atm}} = P_{\text{atm},o}$,
\( P_{1/2} \) is determined:

\[
P_{1/2} = \left( \frac{\Pi_{\text{max}}}{\Pi_0} - 1 \right) (P_{\text{atm},0} - P_{\text{min}})
\] (13)

\( P_{\text{min}} \) is taken here as 0.2 \( P_{\text{atm},0} \), a value within the range for C\(_3\) plants (Black, 1973), C\(_3\) being the most widespread type of carbon fixation. The variability of \( P_{\text{min}} \) affects the \( \Pi (P_{\text{atm}}) \) slope at the lower values of \( P_{\text{atm}} \), but not the overall conclusions below, which depend primarily upon \( \Pi_{\text{max}} \).

The value of \( \Pi_{\text{max}} \) is more difficult to ascertain. To use the WHAK weathering system as a tool to examine the sensitivity of \( P_{\text{atm}} \) and \( T \) to different productivity-CO\(_2\) feedback assumptions, three cases are considered: \( \Pi_{\text{max}} = 2 \Pi_0 \), \( 4 \Pi_0 \), and \( 8 \Pi_0 \). The physical possibility of these productivities will be discussed later. Figure 6 shows the plots of \( \Pi \) versus \( P_{\text{atm}} \) for these three cases.

The function \( \Pi(P_{\text{atm}}) \) from eqs (12) and (13) is combined with eqs (1), (7c), (8c), and (11), and this system's outputs of \( P_{\text{atm}} \) and \( T \) as a function of \( f_{\text{SR}}/f_{\text{A}} \) are shown in figure 7. A and B, compared to the system of eqs (1), (7c), (8c), and (11) used with \( \Pi = \) constant, which produced the curves W3 of figure 5. The response with the variable \( \Pi \) clearly

Fig. 6. Global terrestrial productivity (\( \Pi \)) as a function of atmospheric CO\(_2\) using eq (12) with three different values of the maximum productivity, \( \Pi_{\text{max}} = 2 \Pi_0 \), \( 4 \Pi_0 \), and \( 8 \Pi_0 \).
lowers the sensitivity of $P_{atm}$ and $T$ to the $f_{SR}/f_A$ forcing. It is helpful to examine again the behavior of the pressure buffering-term, $P^*/P_0^*$. Combining eqs (11) and (12), defining $P^*/P_0^* = P_{soil}/P_{soil,o}$, and differentiating with respect to $P_{atm}$,

$$
\frac{d(P^*/P_0^*)}{dP_{atm}} = \frac{\Pi_{max}}{\Pi_o} \left( 1 - \frac{P_{atm,o}}{P_{soil,o}} \right) \frac{P_{1/2}}{(P_{1/2} + P_{atm} - P_{min})^{0.5}} + \frac{1}{P_{soil,o}}
$$

For sufficiently large values of $P_{atm}$ this reduces to $d(P^*/P_0^*)/dP_{atm} \approx P_{soil,o}^{-1}$, which corresponds to the region where $\Pi \approx \Pi_{max}$, and the pressure buffering at this point is exactly that of WHAK case 3 discussed above. By approximating $P_{soil,o}^{-1}$ and $P_{min}$ as zero, and using eq (13) for $P_{1/2}$, we obtain when $P_{atm} = P_{atm,o}$:

$$
\frac{d(P^*/P_0^*)}{dP_{atm}} \approx \frac{1}{1 - \Pi_o/\Pi_{max}}
$$

As $\Pi_{max}/\Pi_o$ increases, the r.h.s approaches $P_{atm,o}^{-1}$, which is exactly that of WHAK case 1. Even when $\Pi_{max}/\Pi_o = 2$, the r.h.s. is significantly larger than $P_{soil,o}^{-1}$. As figure 7 shows when $f_{SR}/f_A$ is larger than about 2, the negative feedback from enhanced productivity maintains $T$ about $1^\circ$, $2^\circ$, and $3^\circ$C lower than the case with $\Pi$ = constant, for the cases of $\Pi_{max} = 2 \Pi_o$, $4 \Pi_o$, and $8 \Pi_o$, respectively. The case with $\Pi_{max} = 2 \Pi_o$ reaches most of its potential cooling when $f_{SR}/f_A$ is greater than about 1.3, while the formulation of $\Pi_{max} = 8 \Pi_o$ reaches its full effects only at much higher values for $f_{SR}/f_A$. As can be seen in figure 6, the three cases reach the same fraction of their potential $\Pi_{max}$ at different $P_{atm}$'s, which accounts for the observed behavior in figure 7.

How realistic are these results? Could the Earth have significantly more productive terrestrial biospheres?

To begin to answer these questions, it is important to realize that the increased continental water supply makes only a small contribution to increased $\Pi$. With the current global mean rainfall of about 700 mm/yr and the value of 2500 mm/yr at which production reaches its maximum, the planet is in an approximately linear regime of the relation between $\Pi$ and rainfall (Leith, 1975). Therefore, neglecting any transpiration-rainfall feedback, $\partial(\Pi/\Pi_o)/\partial(R/R_o) \approx 1$. However, according to eqs (2A) and (7A), the changes in $T$ from figure 7 would increase $R$ at most on the order of tens of percent.

Consider first the feasibility of $\Pi_{max} = 2 \Pi_o$. An average reduction of 30 percent in transpiration per unit biomass for doubled CO$_2$ levels (Idso and Brazel, 1984) would imply that productivity could be 43 percent larger ($0.7^{-1}$) if the supply of water merely remains the same. I estimate that an average potential increase in C$_3$ plant growth from the fertilizing effect of CO$_2$ is on the order of 30 percent (DOE, 1985). These two effects alone could yield $\Pi = 1.86 \Pi_o (1.43 \times 1.30)$. The slightly increased continental water supply (equal to $R$) from either eq
(2A) or (7A) plus the longer growing season would push $\Pi$ somewhat higher. Thus with atmospheric CO$_2$ levels elevated by several times or more, a global productivity double that of today is, at the very least, not unreasonable given what is currently known about maximum plant responses.

In order to have $\Pi \approx 4 \Pi_o$, as required for the $\Pi_{\text{max}} = 8 \Pi_o$ case when $f_{SR}/f_A \approx 2$ (see fig. 7), the mean value of 782 g/m$^2$-yr for the continental production of dry matter would have to increase above the most productive temperate forests (2500 g/m$^2$-yr) and approach the most productive tropical rain forests (3500 g/m$^2$-yr) (all values from Whittaker and Likens, 1975). Since the most productive swamps and marshes have 6000 g/m$^2$-yr, even higher productivities are possible with optimal conditions. However the admittedly limited experiments do not indicate that the effects on enhanced growth from CO$_2$-fertilization and water utilization efficiency will be very much more than the values cited in the above paragraph (DOE, 1985). One example of a high-CO$_2$ limitation occurred in hydroponically-grown wheat, where at 4 times ambient CO$_2$
the reduced water uptake through the roots caused nutrient stress on
the growing plants (Bugbee and Salisbury, 1986). From our knowledge
of the present terrestrial vegetation, and without a several-fold global
increase in rainfall, there is little indication that the productivities
assumed in the $\Pi_{\text{max}} = 8 \Pi_o$ case can be justified.

**CONCLUSION**

The differences in the solutions for paleo-$CO_2$ and paleotempera-
ture shown in this paper indicate the need for further development of
the formulation for weathering in long-term carbon cycle models. The
WHAK river bicarbonate equation (7b) has a structural advantage over
its BLAG counterpart (eq 2b), because it separately resolves the com-
ponent variables of temperature and CO$_2$ pressure. The value of this
separation follows from the experiments on weathering kinetics
(WHAK) and from the need to resolve the roles of atmospheric pCO$_2$
and soil pCO$_2$ in the geochemical carbon cycle. Since biological changes
could concommitantly affect the soil pCO$_2$ at a constant temperature, a
model with a variable terrestrial biosphere requires separate dependen-
cies upon pCO$_2$ and temperature in the weathering rate.

The calculation that combines the WHAK weathering system with
a varying global productivity bears upon the suggestion by Lovelock and
Whitfield (1982) that the biosphere may regulate atmospheric pCO$_2$ and
climate by modulating the weathering rate. They were concerned with
the maintenance of an equable climate, broadly interpreted, since life
began, but little quantitative incorporation of biological processes in
long-term carbon cycle models has been attempted. To provide perspec-
tive for the present results, consider that in response to the geophysical
forcing of spreading rate and continental area changes, a biosphere with
a perfect regulatory capability could operate as a global thermostat and
hold atmospheric pCO₂ and temperature constant.

In this work, the biosphere does not regulate to such ideal
standards. For example, when geophysical forcing drives the planet
toward higher atmospheric pCO₂ and temperature, and when maximum
terrestrial productivity is set to a value double that of today's produc-
tivity (the most defensible possibility for productivity as a function of
atmospheric pCO₂), the resulting temperatures are about 1°C lower
than when productivity is held constant. Higher maximum productivi-
ties stabilize the temperature more, but such high productivities are
difficult to justify. These calculations indicate that the terrestrial bio-
sphere can only moderate, rather than perfectly regulate, the climate
and atmospheric pCO₂ in response to forcing. The possibility of meta-
bolic evolution has been disregarded, as has the suggested role of
organic acids in weathering (WHAK). This study, however, delineates
limitations to the strength of the CO₂-productivity feedback as it
operates through the direct effects upon soil CO₂ and therefore
provides a quantitative beginning for future studies.

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